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Combining Rain Gages With Satellite Measurements for Optimal Estimates of Area-Time Averaged Rain Rates

GERALD R. NORTH

Climate System Research Program, College of Geosciences, Texas A&M University, College Station

SAMUEL S. P. SHEN

Mathematics Department, University of Saskatchewan, Saskatoon, Canada

ROBERT B. UPSON

Climate System Research Program, College of Geosciences, Texas A&M University, College Station

The problem of optimally combining data from an array of point rain gages with those from a low Earth-orbiting satellite to obtain space-time averaged rain rates is considered. The mean square error due to sampling gaps in space-time can be expressed as an integral of a filter multiplied by the space-time spectral density of the rain rate field. It is shown that for large numbers of gages or large numbers of overpasses the two estimates can be regarded as orthogonal in the sense that the optimal weighting is the same as for independent estimators, i.e., the weights are inversely proportional to the error variances that would occur in the single-component case. The result involving point gages and satellite overpasses appears to hold under quite general conditions. The result is interesting since most other design combinations do not exhibit the orthogonality property.

1. Introduction

Precipitation not only has an important effect at the Earth's surface but the corresponding rate of latent heat release aloft is a major driver of the larger-scale atmospheric circulation. This is particularly so of the heavy precipitation in the concentrated convergence zones in the tropics. In global change calculations involving general circulation models (GCMs) of the atmosphere there is a critical lack of such latent heating data for validation and model improvement purposes. Typically, one wants a time series of the rain rate smoothed over grid boxes whose edges are about 500 km and through about one month. This kind of space-time averaging volume is shown in Figure 1. This paper examines the error structure of an observation design relevant to this kind of application.

One never has the luxury of obtaining rain rate data continuously throughout the given space-time volume, but must settle for discretized observations that approximate the space-time integral. For example, an array of rain gages represents a series of rods that run parallel to the time axis of the volume (e.g., Figure 2). Another design is that of a satellite that makes periodic overpasses at a fixed interval of about 12 hours (e.g., Figure 3). This design consists of infinitesimally thin slices of space-time volume perpendicular to the time axis. All of the conceivable designs will have random sampling errors associated with the inherent gaps. In this paper we show the optimum weighting that should be used in combining point surface measurements with those from an idealized low Earth-orbiting satellite. These results should provide some guidance in the optimal estimation of space-time smoothed rain rates and in the ground truth issues associated with satellite-based estimation of rain rates. In the process of solving the problem we find a remarkable orthogonality property between the two designs.

Issues concerning the estimation of climatological time series of precipitation from satellites are hardly academic considering the likelihood of launches of several space-based observing systems such as the Tropical Rainfall Measuring Mission (TRMM [*Simpson et al.*, 1988]) and the Earth Observing System now being studied by NASA [*Baker*, 1990] in the 1990s. A number of preliminary sampling studies for satellite estimating designs have been conducted [*Laughlin*, 1981; *McConnell and North*, 1987; *Bell*, 1987; *Shin and North*, 1988; *Valdés et al.*, 1990; *Bell et al.*, 1990; *Kedem et al.*, 1990; *North and Nakamoto*, 1989]. These studies indicate that at least in the case of tropical convective rain over the oceans, a system like TRMM can limit the sampling errors for 500-km boxes over a month to the order of 10% [*Simpson et al.*, 1988].

We feel that the estimation of space-time averages is sufficiently complicated and confusing that fundamental studies that help us to understand the estimation problem are still useful at this stage. *North and Nakamoto* [1989] (hereinafter referred to as NN) have presented a framework for a large class of studies comparing different measurement designs. Their approach is to make use of the statistical properties of the rain rate field, and exploit the assumed homogeneity and stationarity of the field through the use of the Fourier representation. In this way we can arrive at a compact formula for the mean square error in terms of the space-time spectrum of the rain rate field and a separately factored function which depends only on the design parameters.

The purpose of the present paper is to use the technique to examine a combined sensor problem and explore optimal

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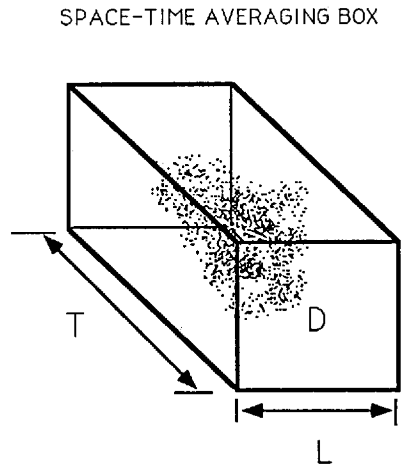

Fig. 1. Schematic diagram of the space-time volume used in the averaging process.

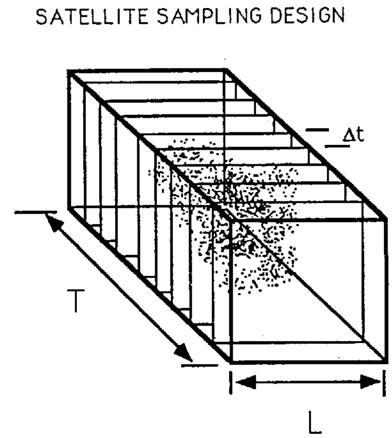

Fig. 3. Design for low Earth-orbiting satellite.

weighting in the context of the NN formalism. In so doing we ignore certain factors such as instrument errors and the fact that the satellite overpasses often only cross a portion of the grid (averaging) box. These are not essential simplifications and at this point their inclusion would only obfuscate the rather clear interpretations that will come from the formulation.

2. Review of North-Nakamoto Approach

North and Nakamoto [1989] have presented a rather general formalism to estimate the mean square errors incurred in estimating space-time means of random fields such as rain rate. They also presented approximate analytical results for two special isolated cases: (1) a satellite making flush visits (we mean by "flush" that the satellite swath during an overpass overlays the grid box completely) at discrete intervals (nominally 12 or 24 hours apart and over a month; the so-called grid (spatial averaging) box is typically 500 km on an edge) and (2) a regular matrix array of rain gages in the box.

We remark that studies of combining rain gages with spatially continuous data have been undertaken by *Creutin et al.* [1988]. We further note that a spectral approach to the estimation of sampling errors with rain gages was conducted by *Rodriguez-Iturbe and Mejia* [1974].

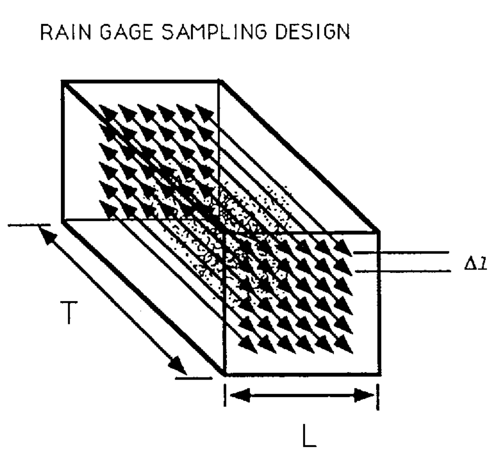

Fig. 2. Design for point rain gages.

2.1. *Mean Square Error Formula*

The NN formalism begins with a random field $\psi(\mathbf{r}, t)$ which represents the instantaneous rain rate at a point $\mathbf{r} = (x, y)$ and at time t. We seek a space-time average over this field

$$\Psi = \frac{1}{L^2T} \int_T \int_D \psi(\mathbf{r}, t)\, d^2\mathbf{r}\, dt \qquad (1)$$

where the integral is to run over the square $-L/2 < x < L/2$; $-L/2 < y < L/2$ and over the interval $0 < t < T$; we refer to the domain

$$D = (-L/2, L/2) \times (-L/2, L/2) \qquad (2)$$

A schematic of the space-time volume is shown in Figure 1.

All attempts at estimating Ψ in practice are hampered by sampling considerations. For example, a single low Earth-orbiting satellite sampling design leads to an estimate given by

$$\Psi_S = \frac{1}{NL^2} \sum_{n=1}^{N} \int_D d^2\mathbf{r}\psi(\mathbf{r}, t_n) \qquad (3)$$

where N is the number of visits the satellite makes in the period T, $t_n = (n - 1/2)\Delta t$ with the interval Δt given by T/N. It can also be written

$$\Psi_S = \frac{1}{TL^2} \int dt \int d^2\mathbf{r}\psi(\mathbf{r}, t)K_S(t) \qquad (4)$$

$$K_S(t) = \Delta t \sum_{n=1}^{N} \delta(t - (n - 1/2)\Delta t) \qquad (5)$$

Similarly, for a rectangular array of rain gages (G), we have an estimator for the area-time average

$$\Psi_G = \frac{1}{TL^2} \int dt \int d^2\mathbf{r}\psi(\mathbf{r}, t)K_G(\mathbf{r}) \qquad (6)$$

where

$$K_G(\mathbf{r}) = (\Delta l)^2 \sum_{n_1,n_2=1}^{M} \delta(x - (n_1 - 1/2)\Delta l)\delta(y - (n_2 - 1/2)\Delta l) \quad (7)$$

and Δl is the spacing in the x and y directions of the gages in the rectangular array. Schematics of the two sampling designs are shown in Figure 2 and 3.

We use as an indicator of error size the mean square deviation of Ψ and Ψ_i taken over an ensemble of realizations

$$\varepsilon_i^2 \equiv \langle(\Psi - \Psi_i)^2\rangle, \qquad i = S \text{ (satellites) or } i = G \text{ (gages)} \quad (8)$$

where the angle brackets denote ensemble averaging. Note that we are evaluating the error squared for a specific month, say March 1990, but the ensemble average is over a large collection of realizations of the field. For all of the present considerations we can adjust the long-term or ensemble mean so that $\langle\Psi\rangle = 0$.

Most of what follows can be expressed best in terms of the Fourier representations of the space-time fields. We define the Fourier transform (FT)

$$\tilde{g}(\nu) \equiv \int_{-\infty}^{\infty} g(x)e^{i2\pi\nu x}\, dx \quad (9)$$

and its inverse

$$g(x) = \int_{-\infty}^{\infty} \tilde{g}(\nu)e^{-i2\pi\nu x}\, d\nu \quad (10)$$

where ν is the wave number (wave cycles per unit length) analogous to frequency f (cycles per unit time).

By inserting the FTs and taking the expectation value, we quickly arrive at the formula (for details, see NN)

$$\varepsilon^2 = \sigma^2 \iiint |H(\boldsymbol{\nu}, f)|^2 S(\boldsymbol{\nu}, f)\, d^2\boldsymbol{\nu}\, df \quad (11)$$

where $S(\boldsymbol{\nu}, f)$ is the space-time power spectral density of the rain rate field defined by

$$\sigma^2 S(\boldsymbol{\nu}, f)\delta^{(2)}(\boldsymbol{\nu} - \boldsymbol{\nu}')\delta(f - f') = \langle\tilde{\psi}^*(\boldsymbol{\nu}, f)\tilde{\psi}(\boldsymbol{\nu}', f')\rangle, \quad (12)$$

$H(\boldsymbol{\nu}, f)$ is a complex-valued design-dependent function, and $\sigma^2 = \langle\psi^2(\mathbf{r}, t)\rangle$ is the point variance of the average of the random rain rate field. The space-time power spectral density $S(\boldsymbol{\nu}, f)$ is a real-valued function. In the above derivation we have assumed throughout that the random rain rate field is statistically homogeneous and stationary.

The formula (11) above neatly separates the problem into two distinct factors under the integral sign: the space-time spectral density of the rain rate field and a design-dependent filter that may be different for each design under consideration. An interesting note is that the formula (11) also tells us that the mean square error depends only upon second moment statistics of the field. There is no assumption being made about the probability density function (pdf) for the field. A highly nongaussian pdf (and rain rates certainly are) may make estimation of the spectral density difficult, but it does not affect the mean square errors in the estimation of the space-time mean for a particular month in a particular grid box once the spectral density is known.

2.2. *Satellite Design*

Consider now the special case of a periodic satellite visitor. NN derived the filter $H(\boldsymbol{\nu}, f)$ for this case:

$$|H_S(\boldsymbol{\nu}, f)|^2 = G(\nu_1 L)^2 G(\nu_2 L)^2 G(fT)^2 \left(1 - \frac{1}{G(f\Delta t)}\right)^2 \quad (13)$$

where we have adopted the notation

$$G(x) = \frac{\sin(\pi x)}{\pi x} \quad (14)$$

In the limiting case of a large number of visits the filter assumes the convenient form

$$|H_S(\boldsymbol{\nu}, f)|^2 \approx G(\nu_1 L)^2 G(\nu_2 L)^2 \left[\frac{1}{T} \sum_{n\neq 0, n=-\infty}^{\infty} \delta\left(f - \frac{n}{\Delta t}\right)\right] \quad (15)$$

which is a Dirac comb along the frequency axis (see NN for graphics). The teeth are at multiples of the satellite revisit frequency; hence, large values of the spectral density at these frequencies will lead to large errors. These can be particularly troublesome for Sun-synchronous satellites which return at the semidiurnal cycle frequency [cf. *Shin et al.*, 1990].

2.3. *Periodic Array of Rain Gages*

For the periodic rectangular array of rain gages in the large number of gages limit,

$$|H_G(\boldsymbol{\nu}, f)|^2 = \frac{G(fT)^2}{L^2} \sum_{n_1,n_2\neq 0} \delta\left(\nu_1 - \frac{n_1}{\Delta l}\right)\delta\left(\nu_2 - \frac{n_2}{\Delta l}\right) \quad (16)$$

which is a two-dimensional Dirac comb (Dirac dog brush) in the (ν_1, ν_2) plane. NN were able to show that for a highly idealized space-time spectral density the random sampling errors for these designs are of the order of 10% separately. The idealized spectrum was tuned to data from the Global Atlantic Tropical Experiment (GATE). The value of 10% occurred when the spacing of gages was Δl = 40 km. The spectrum shape used seems to be substantiated by comparison of the spectra subsequently estimated directly from rain rate data [*Nakamoto et al.*, 1990]. On the other hand, rain rate spectra over land are likely to have characteristics that lead to larger sampling errors for both satellites and point gages [e.g., *Seed and Austin*, 1990].

3. Combination Designs

Now consider the combination of Ψ_S and Ψ_G, a situation that might be called into play in applications over the land surface where rain gages will be available to combine with satellite measurements. The combination may be necessary to reduce the satellite sampling and instrumental errors since over land the satellite must rely either on radar which has a narrower swath (and hence larger sampling errors) or on the use of empirically based methods of rain rate retrieval from microwave radiometers which carry a large instrumental uncertainty. Consider as an estimator the linear combination

$$\Psi_A = \alpha_S \Psi_S + \alpha_G \Psi_G \tag{17}$$

$$1 = \alpha_S + \alpha_G \tag{18}$$

where the coefficients α_i are weights that are to be determined later by minimizing ε^2.

Then we can write for the mean square error

$$\varepsilon^2 = \langle (\Psi - \Psi_A)^2 \rangle \tag{19}$$

$$\varepsilon^2 = \langle ((\alpha_S + \alpha_G)\Psi - (\alpha_S \Psi_S + \alpha_G \Psi_G))^2 \rangle \tag{20}$$

which can be written

$$\varepsilon^2 = \alpha_S^2 \varepsilon_S^2 + \alpha_G^2 \varepsilon_G^2 + 2\alpha_S \alpha_G \varepsilon_{SG}^2 \tag{21}$$

where the ε_S^2 and ε_G^2 are the same as before and the new "interference" term has as a factor

$$\varepsilon_{SG}^2 = \sigma^2 \int d^2\boldsymbol{\nu} \int df\, S(\boldsymbol{\nu}, f) G(\nu_1 L)^2 G(\nu_2 L)^2 G(fT)^2 \cdot \left(1 - \frac{1}{G(f\Delta t)}\right)\left(1 - \frac{1}{G(\nu_1 \Delta l) G(\nu_2 \Delta l)}\right) \tag{22}$$

After some rearrangements we may write

$$\varepsilon_{SG}^2 = \sigma^2 \frac{1}{NM^2} \int d^2\boldsymbol{\nu} \int df\, S(\boldsymbol{\nu}, f) K_{SG}(\boldsymbol{\nu}, f) \tag{23}$$

where

$$K_{SG}(\boldsymbol{\nu}, f) = \frac{\sin^2 (N\pi f \Delta t)}{N(\pi f \Delta t)\, \sin (\pi f \Delta t)} \cdot [G(f\Delta t) - 1] \frac{\sin^2 (M\pi \nu_1 \Delta l)}{M(\pi \nu_1 \Delta l)\, \sin (\pi \nu_1 \Delta l)} \cdot \frac{\sin^2 (M\pi \nu_2 \Delta l)}{M(\pi \nu_2 \Delta l)\, \sin (\pi \nu_2 \Delta l)} \cdot [G(\nu_1 \Delta l) G(\nu_2 \Delta l) - 1] \tag{24}$$

One can show that

$$\frac{\sin^2 (N\pi f \Delta t)}{N(\pi f \Delta t)\, \sin (\pi f \Delta t)} \sim \frac{1}{\Delta t} \delta(f) + \frac{2\pi}{N^2} \sum_{n \neq 0, n = -\infty}^{\infty} \frac{(-1)^{n+1}}{f} \delta'\left(f - \frac{n}{\Delta t}\right) \qquad N \to \infty. \tag{25}$$

The last formula is illustrated in Figure 4 for $N = 10$ and $\Delta t = 0.5$. This convergence to the Dirac delta function and its derivatives is seen to be quite rapid. In the practical case, N is closer to 60 and is already satisfactorily large. It is important to note that another factor in the kernel $K_{SG}(\boldsymbol{\nu}, f)$ vanishes at $f = 0$, namely, $[G(f\Delta t) - 1]$, which is plotted in Figure 5. Hence, when the two factors $\sin^2 (N\pi f\Delta t)/[N(\pi f\Delta t) \sin (\pi f \Delta t)]$ and $[G(f\Delta t) - 1]$ are multiplied, the product essentially vanishes in intervals $(n/\Delta t, (n+1)/\Delta t)$ of f for a large N, $n = \pm 1, \pm 2, \pm 3, \cdots$. At the point $n/\Delta t$, the product is essentially the first derivative of the Dirac delta function for a large N, $n = \pm 1, \pm 2, \pm 3, \cdots$.

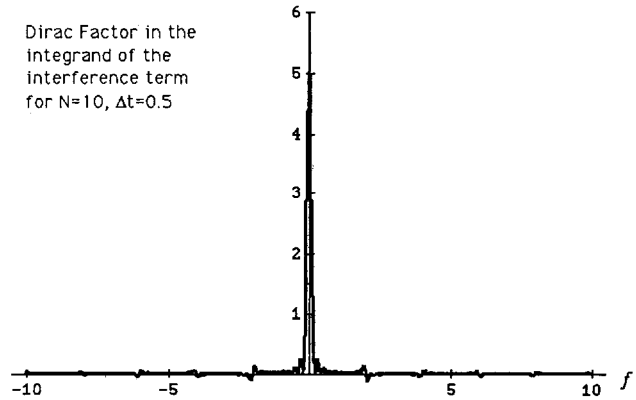

Fig. 4. Plot of the function $\sin^2 (N\pi f\Delta t)/N\pi f \sin (\pi f \Delta t)$ for $N = 10$ and $\Delta t = 0.5$. This function approaches the Dirac delta function $\delta(f)$ as $N \to \infty$.

Explicitly, the convergence result of the product is expressed as follows:

$$\sin^2 (N\pi f\Delta t)/[N(\pi f \Delta t) \sin (\pi f\Delta t)][G(f\Delta t) - 1] \sim \frac{2\pi}{N^2} \sum_{n\neq 0, n = -\infty}^{\infty} \left(1 - \frac{\sin (\pi f \Delta t)}{\pi f \Delta t}\right) \frac{(-1)^n}{f} \cdot \delta'\left(f - \frac{n}{\Delta t}\right) \qquad N \to \infty. \tag{26}$$

Let $A(f)$ be a smooth function of f. Then,

$$\int_{-\infty}^{\infty} df\, A(f) \sin^2 (N\pi f\Delta t)/[N(\pi f\Delta t) \sin (\pi f \Delta t)] \cdot [G(f\Delta t) - 1] \approx \frac{\pi}{N^2} \sum_{n\neq 0, n = -\infty}^{\infty} \cdot \left[(-1)^n \frac{2\Delta t}{n} A'\left(\frac{n}{\Delta t}\right) - \left(\frac{\Delta t}{n}\right)^2 A\left(\frac{2n}{\Delta t}\right)\right] \qquad N \to \infty. \tag{27}$$

A similar conclusion applies to the factors in the kernel $K_{SG}(\boldsymbol{\nu}, f)$ that depend separately on ν_1 and ν_2 when M is

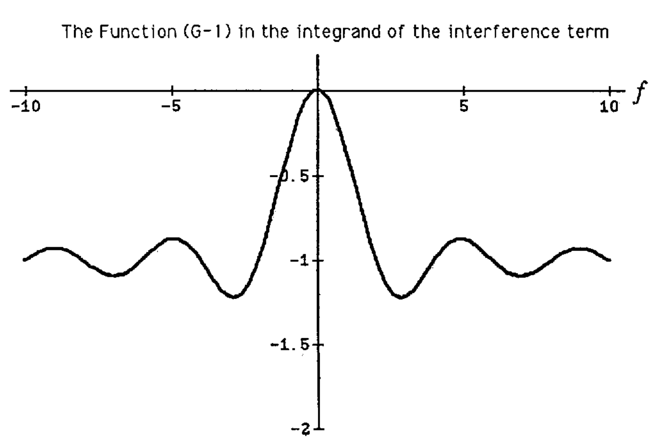

Fig. 5. Plot of the function $G(\Delta t f) - 1$, which vanishes at $f = 0$.

large. Furthermore, the alternation of signs shown in (27) results in some cancellation. Therefore, for any reasonably behaved spectral density function $S(\boldsymbol{\nu}, f)$, the integration

$$\varepsilon_{SG}^2 = \frac{1}{NM^2} \int d^2\boldsymbol{\nu} \int df S(\boldsymbol{\nu}, f) K_{SG}(\boldsymbol{\nu}, f)$$

is vanishing at a rate at least $N^{-3}M^{-6}$ for large N and M. We refer to this phenomenon of vanishing cross term as "design orthogonality."

In actual practical cases, the value of ε_{SG}^2 is already vanishingly small when $M = 5$ and $N = 60$. As an example, let us take $L = 500$ (km), $\Delta l = 100$ (km), $T = 2$ (month), $\Delta t = 12$ (hour), $M = 5$ and $N = 60$. We use the spectral density function found in NN,

$$S(\boldsymbol{\nu}, f) = \frac{\alpha}{4\pi^2\tau_0^2 f^2 + (1 + 4\pi^2\lambda_0^2\nu^2)^2} \tag{28}$$

where $\tau_0 = 12$ (hour), $\lambda_0 = 40$ (km) and α is a normalization constant which satisfies the following condition:

$$\alpha\sigma^2 = 2\tau_0 L^2 \sigma_A^2. \tag{29}$$

Here σ_A^2 is the variance of the spatial average of the random field $\psi(\mathbf{r}, t)$. With these data, $(\varepsilon_{SG}^2/\sigma_A^2)^{1/2} < 1.0 \times 10^{-5}$, whereas the noninterference terms $(\varepsilon_{S,G}^2/\sigma_A^2)^{1/2}$ separately are of the order of 10^{-1}.

Now consider the values of α_S and α_G (sum equal to 1) which minimize the value of the mean square error ε^2. This is very easily computed by setting the derivative equal to zero, leading to the optimal weights

$$\hat{\alpha}_S = \frac{\varepsilon_G^2}{\varepsilon_G^2 + \varepsilon_S^2} \tag{30}$$

$$\hat{\alpha}_G = \frac{\varepsilon_S^2}{\varepsilon_G^2 + \varepsilon_S^2} \tag{31}$$

In other words, the estimate from each sensor design is weighted inversely according to the error variance due to that design alone. A component that has a small error variance will be weighted very strongly and conversely. This is a familiar result when one is combining estimates of a mean from estimators that are independent of one another. The design orthogonality for the case of satellite–rain gage combinations is not altogether trivial, since it does not obtain for most combinations such as combining data from a second satellite with a different phase or period to data from a given one. Nor would it occur in augmenting an array of rain gages by another set intermingled with the first.

4. Conclusions

We have demonstrated that under rather general conditions the satellite overpass and the dense array point rain gage designs for estimating area-time averages of rain rate are orthogonal, i.e., the mean square error for the combination is just a linear combination of the mean square errors for the individual estimators with the coefficients inversely proportional to the error variances that would occur without the other design being present. This means that the two estimates are essentially independent or that the random sampling errors from one design are uncorrelated with those from the other ($\langle(\Psi - \Psi_G)(\Psi - \Psi_S)\rangle = 0$) and combining the two can be done just as well after the fact. Ordinarily, in combining two sensors such as a second satellite, the random sampling errors from one sensor to the other will be positively correlated, leading to a positive contribution to the total mean square error for the combination. Adding a second sensor helps but adding one that is uncorrelated with the first helps most.

Several assumptions made in the proof above can be removed. For example, the homogeneity assumption can be removed by using the Karhunen-Loeve basis set in D instead of using the Fourier basis. These functions (variously called empirical orthogonal functions or EOFs in the meteorological literature [e.g., *North et al.*, 1982] are the eigenfunctions of the covariance matrix on a fine grid in the domain. Clearly, there is also nothing special about the periodicity of the rain gage or satellite overpasses in the designs used here. The important point is that the sampling errors due to spatial gaps are uncorrelated with those due to the temporal gaps. In our study of the satellite design we have assumed that all visits are flush. Relaxing this assumption leads to larger sampling errors for the satellite sampling design but does not change the design orthogonality conclusion reached here.

It appears that if the separate sampling errors due to the two designs studied here are comparable, it is desirable to pool the data and get a significant reduction in the overall random sampling error. If they are exactly equal there will be a halving of the mean square error or a reduction of the rms error of a factor of $1/\sqrt{2}$. If the sampling errors for one system are much larger than for the other, inclusion of the cruder data will not help very much. The studies by NN for monthlong averages over 500-km boxes suggest that the gage separations would have to be of the order of 40 km apart to achieve this kind of parity based upon tropical oceanic data.

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G. R. North and R. B. Upson, Climate System Research Program, College of Geosciences, Texas A&M University, College Station, TX 77843.

S. S. P. Shen, Mathematics Department, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 0W0.

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