Alberta Agriculture, Food and Rural Development

Statistical Analysis of Drought Indices and Alberta Drought Monitoring

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Table of Contents

Summary 2
1. Introduction
2. Background and Literature Review
3. Data
4. Methodology
4.1 Standardized Precipitation Index (SPI)
4.2 Rainfall Anomaly Index (RAI) 10
4.3 Rainfall Decile Index (RDI)
4.4 Standardized Anomaly Index (SAI) 12
4.5 Principal Component Index (PCI) 13
4.6 Optimal Index 15
5. Alberta Historical Drought Record 16
6. The Threshold for Drought Categories
7. Wheat Drought in Canada's Palliser Triangle
8. Probability Transition of Weekly Precipitation
9. The Gamma Fitting Problem
10. Conclusions and discussion
11. Acknowledgements
References

Summary

This report includes a statistical analysis of six drought indices for monitoring Alberta drought events from 1901 to 2000. The data used are the interpolated daily precipitation data on the 149 ecodistrict polygons (EDP) over Alberta. The analyzed indices are standardized precipitation index (SPI), rainfall anomaly index (RAI), rainfall decile index (RDI), standardized anomaly index (SAI), principal component index (PCI), and optimal index (OPI). The historically documented drought records of five sites (Beaver Lodge, Lacombe, Lethbridge, Vegreville, and Swift Current [in Saskatchwan]) are classified into drought categories D4, D3, ..., D0, and wet categories –D1, -D2, and –D3. The thresholds of the drought categories for different indices are calculated. The wheat drought of Canada's Palliser Triangle was used as a validation analysis of the drought indices. The transitional probability of drought categories from one week to the next is calculated. Some discussions on the theory of calculating SPI are included. It has been found that the while all the drought indices are highly correlated with precipitation, the PCI has the highest correlation. The transitional probability analysis for the south Alberta agricultural region shows that the chance of transition from normal to extremely dry is highest in the mid May, hence this region's spring seeding is extremely vulnerable to precipitation and an effective irrigation system is of great importance to the early stages of crop development.

1. Introduction

Recurring droughts in Alberta can seriously decrease crop yields and thus harm the whole Alberta agriculture industry. As a result of the drought events in Alberta such as those of the Dirty Thirties and the 1980s and 1990s, the Alberta government has implemented some soil-conservation and water-management programs. The last three years' successive droughts in Alberta resulted in payments of over \$1 billion Canadian dollars for farm assistance from the Alberta government and also in an Agriculture Drought Risk Management Plan for Alberta in 2002 (Source: Alberta Premier's web site). An effective drought-monitoring system and a reliable drought-monitoring scheme are urgently needed in order to properly assess the drought severity and predict future drought events and the duration of current droughts.

This project is an integral part of the Alberta agriculture drought risk management plan. Through analyzing various types of drought indices, quantitative information will be provided on the severity of the drought conditions from various perspectives, including the meteorological, hydrological, agricultural, and social-economic aspects. The analyzed information will be integrated into an operational system that monitors the Alberta agricultural drought. The system will help with the optimal management of the drought risks for Alberta agriculture.

Although various drought indices exist, their applicability is specific for both purposes and regions. For example, the Palmer Index was designed specifically for semiarid and dry subhumid climates and did not take into account the contributions of snowmelt to surface water. (Snowmelt is common in Alberta.) Therefore, to properly assess the drought conditions for Alberta, a new index should be developed.

The specific purposes of this project are (i) to examine several meteorological drought indices and (ii) to develop an optimal index through a weighted average. The weights will be determined by the least square approach. The new index is expected to give a more accurate assessment of drought conditions. Both the spatial and temporal spectral properties of the Alberta drought events will be explored via principal component analysis and wavelet analysis. A thorough understanding of the dynamic mechanism of the drought events will be provided through the analysis of global atmospheric circulation and local microclimatic conditions. Eventually, an operative drought-monitoring system will be developed in collaboration with Alberta Agriculture, Food and Rural Development.

2. Background and Literature Review

Despite some irrigation programs, Alberta crops, particularly the spring wheat, are vulnerable to drought conditions. The 2001 drought made the Alberta yield lower than that of each of the 10 proceeding years, for the 2001 yield was only 84% of the ten-years' mean (Source: Agriculture Division, Statistics Canada). Hence, the proper assessment and prediction of drought severity and duration will be very helpful for reducing the effects of droughts on the Alberta agriculture industry.

Hall et al. (2003) composed a document on the history of agricultural droughts in Alberta. The document includes the records and anecdotes reporting drought on prairies during the 20th century. Although these data are valuable in validating a droughtmonitoring model, they do not give a systematic assessment of the drought severity; yet using dimensionless indices to classify drought conditions is an effective way to monitor moisture deficiency and the duration and intensity of a drought over an entire region. Many indices have been proposed. From the earlier indices that directly measure the rainfall deficiency to the more complicated Palmer Drought Severity Index, Surface Water Supply Index and Standardized Precipitation Index, drought indices have evolved slowly during the last two centuries (Heim, 2002; Keyantash and Dracup, 2002). Usually, meteorological drought-monitoring programs use the stations' observation data, but the non-uniform distribution of the stations makes the reliability questionable. It is advantageous to develop the drought indices according to the land areas with similar ecological properties or soil properties. This consideration motivated us to use the interpolated precipitation data of the Ecodistrict Polygons (EDP) for drought index-computing and monitoring. Shen et al. (2001) developed a hybrid method that interpolates the daily station precipitation data onto the EDPs. The interpolation retains both the spatial and temporal variances of the original precipitation field. Specifically, it retains the precipitation frequency of a region over a given period, say, a month. This data set of daily EDP precipitation allows one to study the precipitation indices and the probability of a change from one drought condition to another at different time scales, ranging from a week to 36 months.

In this research, we will analyze five kinds of commonly used meteorological drought indices and optimally combine them to monitor the drought events, particularly the wheat drought, in Alberta. The wheat-drought events in southern Alberta, ranked according to their strength in descending order, occurred in 2002, 2001, 2000, 1984, 1936, 1977, 1985, 1961, 1937, 1943, and 1931. The drought events of the "Dirty Thirties," the 1980s, and the last three years are used to validate the applicability of the drought indices for drought assessment.

3. Data

One hundred years of daily precipitation data from January 1, 1901 to December 31, 2000 are used for this study. The stations' observation data are interpolated onto 149 Alberta ecodistrict polygons (EDP). The interpolation method used is a hybrid of the methods of inverse-distance-weight and nearest-station-assignment developed by Shen et al. (2001). The interpolated daily data fit not only the climate mean but also climate variability, in particular, the number of days with precipitation per month. This method also can reliably calculate the precipitation amount for a day over an EDP. The 2001-2002 station data of precipitation and their interpolation are also used in this investigation.

Alberta is divided into 10 ecoregions (Fig. 3.1) according to distinctive regional ecological characteristics including climate, physiography, vegetation, soil, water and

fauna. Each ecoregion contains a number of EDPs. Our indices analysis is conducted on the ecoregions and especially focuses on the southwest corner of Alberta.

The1912-2001 wheat-yield data of Lethbridge and the 1898-1969 wheat-yield data of Saskatchewan are available. These data are helpful for investigating the historical drought conditions and for validating the accuracy and effectiveness of our indices when used as an agricultural drought monitor.

A descriptive document on the history of agricultural droughts in Alberta during the 20th century was composed by Hall et al. (2003). The records are further digitalized for statistical analysis.



Fig. 3.1 Alberta ecoregions

Hydrological data set HYDAT from Environment Canada will be used. It contains daily, monthly, and/or instantaneous information for streamflow, water level, suspended sediment concentration, sediment particle size, and sediment load data for over 2900 active stations and some 5100 discontinued sites across Canada. This dataset can be used to assess the water balance over Alberta and is helpful for us to derive the optimal index.

4. Methodology

Our research approach is divided into two steps. First, we examine the existing indices that are likely applicable to Alberta. Second, we optimally combine them to form a new index. The computational methods and comments relating to the indices are presented below. Although the computational methods used in this research follow the existing literature, our in-depth investigation of some indices like the Principal Component Index (PCI) appears to be new in the context of drought monitoring. The advantage of using the PCI is two-fold. First, based on our preliminary numerical results, it is more sensitive than other indices to drought events. Second, the PCI is related to the spatial patterns of precipitation and can be incorporated into climate dynamics to explain certain spatial characteristics of drought events.

4.1 Standardized Precipitation Index (SPI)

Calculation of the SPI for any location is based on the long-term precipitation record for an objective period (3 months, 6 months, etc.). This long-term record is fitted to a probability distribution, which is then transformed into a normal distribution so that the mean SPI for the location and desired period is zero. The precipitation field is usually not in normal distribution, particularly in a short time scale. Various types of precipitation distributions have been used for different spatial regions and different time scales. The SPI is defined as the equivalence value of the accumulative probability in normal distribution (McKee et al., 1993)(Fig. 4.1). The computational procedure follows three steps. First, the precipitation time series data are fitted to a 2-parameter gamma distribution, whose two parameters are estimated by the maximum likelihood method (Thom, 1958). The probability density function of the gamma distribution is

$$f(X \mid a,b) = \frac{1}{b^{a}\Gamma(a)} x^{a-1} e^{-\frac{x}{b}},$$

where a(>0) is a shape parameter, $\beta(>0)$ is a scale parameter, and

$$\Gamma(a) = \int_0^\infty y^{a-1} e^{-y} dy$$

is the gamma function.

Second, after the estimation of the parameters, the probability of each precipitation observation can be calculated from the gamma distribution with the two parameters by using the gamma cumulative distribution function. The cumulative probability is given by

$$F(X \mid a, b) = \int_{0}^{x} f(x) dx = \frac{1}{b^{a} \Gamma(a)} \int_{0}^{x} t^{a-1} e^{\frac{t}{b}} dt.$$

Because the precipitation total could be zero for some time scales, and the gamma function is undefined when x = 0, the cumulative probability could be revised as

$$H(x) = q + (1-q)F(x),$$

where q is the probability of a zero. It can be estimated by $\frac{m}{n}$, where m is the number of zeros in the time series, and n is the total number of observations.

Finally, the inverse of the normal cumulative distribution function with mean $\mu = 0$ and variance $\sigma = 1$ at the corresponding probability can be calculated for each observation. These resulting values are the SPI's.



Fig. 4.1 Schematic diagram of an equiprobability transformation from a fitted gamma distribution to the standard normal distribution (from Lloyd-Hughes and Saunder, 2002)

An important feature of this equi-probability transformation from one distribution to another distribution is that the probability of being less than a given value of a variate is the same as the probability of being less than the corresponding value of the transformed variate.

The SPI can produce not only monitoring information of index values but also the information of probability, percent of average, and precipitation deficit during drought. Positive SPI values indicate greater than median precipitation, while negative values indicate less than median precipitation. The magnitude of departure from zero represents the probability of occurrence so that decisions can be made based on this SPI. The precipitation used in the SPI can be used to calculate the precipitation deficit for the current period and to calculate the current percent of average precipitation for the time period under study (McKee et al., 1993; Hayes et al., 2000).

The SPI can be calculated for a variety of time scales and for different water variables such as soil moisture, ground water, snow-pack, reservoir, and streamflow. This feature allows the SPI to monitor both short-term and longer-term water resources. Since the precipitation data are transformed to a normal distribution, the SPI allows comparison between different locations. Guttman (1998) compared the historical time series of the Palmer Drought Index with the time series of the corresponding SPI through spectral analysis and indicated that the SPI is spatially consistent (invariant) and easily interpreted.

Although the SPI has the strengths mentioned above, it also has some limitations. Hayes et al. (2000) stated that the SPI is only as good as the data used in calculation. Before the SPI is applied in a specific situation, knowledge of the climatology for that region is necessary. For short time scales, the SPI is similar to the percent of normal representation of precipitation, which can be misleading in regions with low seasonal precipitation totals. Guttman (1999) pointed out that at least 50 years of data are needed for drought periods of 1 year or less and that SPIs with time scales longer than 24 months may be unreliable.

The SPI is now used by the Colorado Climate Center, the Western Regional Climate Center, and the National Drought Mitigation Center of the United States to monitor drought conditions.

4.2 Rainfall Anomaly Index (RAI)

The RAI was developed by van Rooy (1965). The positive and negative RAI indices are computed by using the mean of ten extremes. Let \overline{M} be the mean of the ten highest precipitation records for the period under study, \overline{P} the mean precipitation of all the records for the period, and P the precipitation for the specific year. Then the positive RAI (for positive anomalies) for that year is

$$RAI = 3\frac{P - \overline{P}}{\overline{M} - \overline{P}}$$

Let \overline{m} be the mean of the ten lowest precipitation records for the period under study. Then the negative RAI (for negative anomalies) for that year is

$$RAI = -3\frac{P - \overline{P}}{\overline{m} - \overline{P}}$$

RAI	Class description
≥ 3.00	Extremely wet
2.00 to 2.99	Very wet
1.00 to 1.99	Moderately wet
0.50 to 0.99	Slightly wet
0.49 to -0.49	Near normal
-0.50 to -0.99	Slightly dry
-1.00 to -1.99	Moderately dry
-2.00 to -2.99	Very dry
≤ - 3.00	Extremely dry

The classification of the index used by van Rooy (1965) is as follows.

4.3 Rainfall Decile Index (RDI)

The RDI is defined by dividing the distribution of occurrences over a long-term precipitation record into sections for each ten percent of the distribution. Each of these categories is called a "decile." The first decile is the rainfall amount not exceeded by the lowest 10% of the precipitation occurrences. The second decile is the precipitation amount not exceeded by the lowest 20% of occurrences. The fifth decile is considered the median and is the precipitation amount not exceeded by 50% of the occurrences over the period of record (Hayes, 2000). The deciles are grouped into five classifications according to a decile's departure from the normal condition (Gibbs and Maher, 1967).

Deciles 1-2	lowest 20%	much below normal
Deciles 3-4	next lowest 20%	below normal

Deciles 5-6	middle 20%	near normal
Deciles 7-8	next highest 20%	above normal
Deciles 9-10	highest 20%	much above normal

A region is considered to be "drought affected" if the total precipitation of the preceding three months falls within the lowest decile of the historical distribution (Kinninmonth et al., 2000). The conditions end when either of the following happens:

- (1) The precipitation of the past month places the following three-month total in or above the fourth decile.
- (2) The total precipitation for the past three months is in or above the eighth decile.

The advantages of the RDI are that (1) it is simple to compute, and (2) it requires less data and fewer assumptions than the Palmer Drought Severity Index. Unlike the SPI, the RDI computation has no assumption about the precipitation distribution. As stated by Hayes (2000) and Keyantash and Dracup (2002), the RDI has some limitations. It requires a long climatological record to calculate the deciles accurately. The two criteria to indicate the end of the "drought affected" condition can lead to conceptual difficulties when the region under study has highly seasonal precipitation.

The RDI is used in the Australian Drought Watch System and appears to be very effective.

4.4 Standardized Anomaly Index (SAI)

In this research, the SAI is defined by

$$SAI(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{R_i(t) - \mu_i}{\sigma_i}$$

where $R_i(t)$ denotes the precipitation for the *i* th EDP and *t* th year, μ_i is the 1961-1990 mean (i.e., climatology) of $R_i(t)$, σ_i is the standard deviation of $R_i(t)$ in 1961-1990, and *N* denotes the total number of EDP polygons inside an ecoregion.

For the monthly time scale, a specific month has to be identified. The index is computed for this month, say, June. The $R_i(t)$ is the June total precipitation for the *i* th EDP from t = 1901 to t = 2000. The mean μ_i and standard deviation σ_i are the values specified for the June data. The June SAI has 100 values, one for each year from 1901 to 2000. An index curve can be drawn accordingly.

Katz and Glantz (1986) stated that the SAI is conceptually the conversion of the observed rainfall at a station into units, the number of standard deviations from the long-term station mean. In order to make comparisons of index values meaningful, it is desirable to have fixed expected value and variance for an index. The SAI meets the first goal of possessing an expected value (i.e., zero) that is invariant under any changes in the locations on which the index is based, but it does not achieve the unit variance.

The SAI can be re-expressed as

$$SAI(t) = c + \frac{1}{N} \sum_{i=1}^{N} w_i R_i(t),$$

where $w_i = 1/\sigma$ and c is a constant given by

$$c = -\frac{1}{N} \sum_{i=1}^{N} \frac{\mu_i}{\sigma}$$

So, the SAI can be viewed as a weighted average or linear combination of the rainfall for the N polygons, with the weights being inversely proportional to the station's standard deviations. Because sites with higher mean rainfall tend to also have higher standard deviations, the SAI results in weighting the drier sites more than the wetter ones (Katz and Glantz, 1986).

4.5 **Principal Component Index (PCI)**

The PCI is the coefficient of the Empirical Orthogonal Function (EOF) when data are decomposed as the product of spatial components (EOFs) and temporal components (PCIs).

Here, we consider only the anomaly data: $a_i(t) = R_i(t) - \mu_i$, where μ_i is still computed from 1961-1990, $a_i(t)$ denotes the precipitation anomaly for the *i*th polygon at time *t*, and $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, T$. The $N \times T$ data matrix A is

$$A = \begin{bmatrix} a_1(1) & a_1(2) & \cdots & a_1(T) \\ a_2(1) & \cdots & \cdots & a_2(T) \\ \cdots & \cdots & \cdots & \cdots \\ a_N(1) & \cdots & \cdots & a_N(T) \end{bmatrix}.$$

The entries in A satisfy

$$\frac{1}{T}\sum_{t=1}^{T}a_{i}(t)=0, \quad i=1,2,\cdots,N.$$

The matrix A can be decomposed into

$$[A] = [E][P],$$

where E is a $N \times N$ matrix (i.e., EOFs)

$$E = \begin{bmatrix} e_1(1) & e_2(1) & \cdots & e_N(1) \\ e_1(2) & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ e_1(N) & e_2(N) & \cdots & e_N(N) \end{bmatrix},$$

and P is the $N \times T$ principal component matrix

$$P = \begin{bmatrix} p_1(1) & p_1(2) & \cdots & p_1(T) \\ p_2(1) & \cdots & \cdots & p_2(T) \\ \cdots & \cdots & \cdots & \cdots \\ p_N(1) & \cdots & \cdots & p_N(T) \end{bmatrix}.$$

The E and P have orthogonal characteristics as follows

$$E_{k}'E_{l} = \sum_{i=1}^{N} e_{k}(i)e_{l}(i) = 0$$
, for $k \neq l$ and
 $P_{k}P_{l}' = \sum_{j=1}^{T} p_{k}(j)p_{l}(j) = 0$, for $k \neq l$.

Here, the prime indicates the transpose of a matrix. So, the principal components can be expressed as

$$[P] = [E'][A].$$

The k th principal component is

$$P_k(t) = \sum_{i=1}^N a_i(t) e_k(i).$$

14

The first principal component PCI1 (P_1) is used as the drought index. The second principal component PCI2 (P_2) is also computed to verify its orthogonality to PCI1.

4.6 **Optimal Index**

The objective of this research is to find an optimal drought index that combines the selected indices so that its ability to assess drought severity and duration will be greater than that of the current indices. Two major challenging questions in this research are (a) how to define the objective drought parameter, and (b) how to verify our results. A proposed optimal average method is to construct an optimal index \hat{I} ,

$$\hat{I} = \sum_{j=1}^{N} w_j I_j ,$$

under the constraint

$$\sum_{j=1}^N w_j = 1,$$

such that the mean square error

$$\varepsilon^2 = \left\langle (I - \hat{I})^2 \right\rangle$$

is minimized. Here I_j is a selected drought index, w_j is the weight, N is the number of indices selected, I is an objective drought parameter that accurately describes the drought condition. This least square procedure needs an accurate target object so that we can compare the estimated value with the true value. Hydrological data will be helpful to construct the "true value," but its accuracy and representation of the drought conditions would still need investigation. Also, to verify if our index captures the drought conditions or not, an accurate drought record is needed. That is, when did the droughts actually occur? For the drought record, we have used some results derived from the water budget. Several of our indices can capture the drought events in the record, but some situations still cannot be explained. Therefore, the question arises: are the hydrological data alone enough to represent the drought conditions? Perhaps more sources are needed to extract the accurate information.

For the optimization method, our research will be based on the idea of Shen et al. (1994), who solved the optimal averaging problem by using the covariance structure of the space in question. The mean square error of the optimal averaging can be explicitly expressed in terms of the sum of the contributions from successive EOF modes. Our problem involving the optimal combination of indices will also be converted into a problem of computing a covariance matrix. Various kinds of difficulties could be encountered in the procedure. Theoretical breakthroughs are expected in this optimization research, which will have significant practical applications.

5. Alberta Historical Drought Record

A historical record of drought in Alberta has been compiled from information derived from Hall et al. (2003) report, consisting of records and anecdotes of drought events in five locations in Alberta and Saskatchewan. The records span the period of 1901-2002, and 5 specific locations (Fig. 5.1): Beaverlodge, Lacombe, Lethbridge, Vegreville (in Alberta), and Swift Current (in Saskatchewan). Records from early in this period and sporadically throughout are difficult to obtain, therefore, this is not a complete historical record and missing data is present. Since the Hall (2003) report is a descriptive history of Alberta drought, the data has to be digitalized for statistical analysis (Table 5.1). In doing so, a drought category is used to describe the drought condition of an area for a particular year. The drought classification adopted is the Drought Severity Classification, currently in use by the US Drought Monitor, administered by the NOAA National Climatic Data Center. The drought classification consists of numerical ratings of 0 to 4. The wet conditions are also classified with three classifications, which have opposite signs of the drought indices. The drought classification is according to three factors: the extent of crop/pasture loss, current fire risk, and the water supply in the area.



Fig. 5.1 Location of the five cities with drought records.

Exceptional Drought, denoted by D4, is characterized by exceptional and widespread crop/pasture losses, exceptional fire risk, and shortages of water in reservoirs, streams, and wells creating water emergencies. Extreme Drought, denoted by D3, is classified by major crop/pasture losses, extreme fire danger, and widespread water shortages or restrictions. Severe Drought, denoted by D2, occurs when crop/pasture losses are likely, the fire risk is high, water shortages are common, and water restrictions are imposed. Moderate Drought, denoted by D1, is characterized by some damage to crops and pastures, a high fire risk, with streams, reservoirs, and wells low or some water shortages developing or imminent, and voluntary water use restrictions requested. Abnormally dry, denoted by D0, is the condition in a period for a region either going into drought or coming out of drought. Going into drought is characterized by short-term dryness, slowing planting and growth of crops or pastures, above average fire risk; Coming out of drought is characterized by lingering water deficits and pastures/crops not fully recovered.

The additions to this classifications system are -D1 to -D3, where -D1 denotes a normal precipitation condition, where there is enough moisture and sunlight for crops, little fire risk and reservoirs and wells have adequate supplies. -D2 represents an abnormal amount of precipitation for the period, more than enough for crops, a low to none fire risk, and full reservoirs and wells. -D3 indicates a severe amount of precipitation, such as floods, or torrential downpours, possibly too much for crops, no fire risk, and possible overflowing of reservoirs.

Year	BeaverLodge	Lacombe	Lethbridge	Vegreville	Swift Current
1901	-99	-2	-2	-1	-2
1902	-99	-2	-2	-99	-2
1903	-99	-2	1	-99	-2
1904	-99	-2	2	-99	-1
1905	-99	0	0	-1	-1
1906	-99	1	-2	0	-2
1907	-99	-1	1	0	-1
1908	-99	-1	1	-99	-1
1909	-99	-1	2	0	-2
1910	-99	0	3	0	2
1911	-99	-2	-2	-1	0
1912	-99	-2	0	-1	-1
1913	-1	0	0	-1	2
1914	-99	-2	1	-2	2
1915	-2	-2	-2	-2	-1
1916	1	-1	-2	-2	-2
1917	-2	0	2	-1	1
1918	0	0	2	-99	2
1919	-2	0	2	1	3
1920	-2	1	3	1	2
1921	-1	0	2	-99	0
1922	1	1	2	1	-1
1923	1	-2	-1	-99	-2
1924	-1	-2	-1	1	-1
1925	-2	-1	-2	-99	1
1926	0	-2	1	0	-1
1927	-2	-2	1	-99	-2
1928	0	0	1	-99	0

Table 5.1 History of Albe	erta drought: classified	d using the NOAA	NCDC drought
class	ification (-99 indicates	missing data)	

1929	-2	1	1	1	1
1930	0	-1	1	1	0
1931	0	-2	2	2	1
1932	0	-2	0	-99	2
1933	-2	0	1	-99	2
1934	-2	1	2	2	3
1935	-2	-2	3	2	3
1936	-2	3	4	3	4
1937	-1	2	0	3	4
1938	2	2	2	2	2
1939	-2	0	3	-99	2
1940	0	-1	2	-99	2
1941	-2	-2	1	-99	2
1942	0	-2	-2	-3	-2
1943	0	1	1	-3	2
1944	0	-2	1	-99	-2
1945	0	-2	-2	-99	1
1946	2	-2	-2	-99	-2
1947	-2	-2	-2	-99	1
1948	0	-2	-2	-3	0
1949	0	2	0	-3	1
1950	0	2	1	0	1
1951	-2	-2	-2	-3	1
1952	1	-1	0	-99	-2
1953	-99	-2	-2	-99	1
1954	-99	-2	-1	-2	-1
1955	-99	-2	-2	-99	-99
1956	-99	-2	-2	-99	-99
1957	-99	0	-2	1	-99
1958	0	0	1	0	-99
1959	0	0	-99	0	-99
1960	-99	-99	-99	-99	-99
1961	0	1	3	3	3
1962	-2	2	2	-1	-99
1963	1	1	-99	0	-99
1964	-2	0	2	0	-99
1965	-2	-2	-99	-1	-99
1966	-2	-99	-99	0	-2
1967	-1	-99	2	-99	1
1968	-99	-99	2	1	1
1969	-99	0	-99	-99	-1
1970	0	-99	2	2	-1
1971	-99	0	-99	-99	0
1972	-99	-1	-99	-99	0
1973	-2	-1	1	-1	2

1974	-99	-1	0	-3	0
1975	0	-99	0	-99	0
1976	-99	-1	1	-99	0
1977	-99	-99	2	-99	2
1978	-99	-1	-2	-99	-99
1979	-99	-99	-1	-99	-99
1980	-99	-1	0	-99	2
1981	-99	-2	0	-99	2
1982	-99	-2	0	-99	-99
1983	-99	0	0	-99	-99
1984	-99	-1	3	-2	2
1985	-99	0	2	-1	2
1986	-99	-99	0	-99	-99
1987	-99	-99	1	-99	2
1988	-99	-1	3	-2	2
1989	-99	0	-2	-2	-99
1990	-99	-2	-99	-1	-99
1991	-99	-2	1	0	-99
1992	-99	0	3	-99	2
1993	-99	-99	-99	-99	-99
1994	-99	-99	-99	-99	-99
1995	-99	-99	-99	-99	-99
1996	-99	-99	-99	-99	-99
1997	-99	-99	0	-99	-99
1998	-99	-99	1	-99	-99
1999	-99	-99	1	-99	-99
2000	-99	3	2	3	2
2001	-99	3	3	3	3
2002	-99	4	4	4	4

The quantitative records can only give the yearly information due to the lack of detailed description for monthly or seasonal drought conditions. It can be improved when more information is available. Table 5.2 gives the number of years with each of the drought category for the five cities. The results in this table are derived from the data in Table 5.1. Please note that the Total in the last column is the total number of years with records, since there are many missing data in Table 5.1. Figs. 5.2- 5.6 show the temporal variation of the drought conditions for the five cities during the 102-year period, where the dots above the x-axis indicate the dry years and the dots below the x-axis indicate the wet years. It can be seen that Lethbridge and Swift Current are relatively drier than other cities, while Beaverlodge and Lacombe are relatively wetter.

	D4	D3	D2	D1	D0	-D1	-D2	-D3	Total
Beaverlodge	0	0	2	5	19	5	20	0	51
Lacombe	1	3	5	8	20	16	32	0	85
Lethbridge	2	9	19	21	15	4	19	0	89
Vergreville	1	5	5	8	12	11	7	6	55
Swift Current	3	5	22	12	10	12	13	0	77

Table 5.2 Number of dry and wet years of the five cities from 1901 to 2002



Fig. 5.2 Historical drought conditions of Beaverlodge.



Fig. 5.3 Historical drought conditions of Lacombe.



Fig. 5.4 Historical drought conditions of Lethbridge.



Fig. 5.5 Historical drought conditions of Vegreville.



Fig. 5.6 Historical drought conditions of Swift Current.



Fig. 5.7 Historical drought records at the five cities in (a) 1915, (b) 1936, (c) 1951, and (d) 1961.

Fig. 5.7 demonstrates the spatial distribution of historical drought conditions for the year 1915, 1936,1951 and 1961. In the four years, 1915 and 1951 were wet years; and 1936 and 1961 were dry years. It can be seen that in these years, the southern part of the Alberta province was drier than the northern part. In the dry years, the drought condition of Beverlodge was not as sever as the southern cities and even appeared to be wet in 1936.

6. The Threshold for Drought Categories

Drought is classified into 5 categories as described in the last section. This section discusses the threshold for drought categories, i.e., the drought triggers. The principal basis for the threshold is the probability of occurrence. The exceptional drought D4 is an event once in 50 years, extreme drought D3 once in 20 years, severe drought D2 once in

10 years, moderate drought D1 once in 5 years, and abnormally dry D0 once in 3 years. Because of this basis, the probability-based drought indices are easy to use for drought interpretation. Of the five indices introduced in Section 4, only the SPI and RDI take into account the probability of occurrence. The classification of other indices, RAI, SAI and PCI, are either arbitrary or not clear at all. This section will analyze all the drought indices in the probability sense and develop methods for calculating the threshold values when an index is not probability-based, such as the PCI.

The percentile-based indices, such as percent of normal precipitation, are explicit probability-based indices and do not need further clarification. However, the probabilitybased SPI still needs a drought classification. SPI is calculated from fitting a probability distribution model and being compared for the equivalent probability with a standard normal distribution. The fitting part is often problematic and the major source of error. The probability interpretation of the SPI trigger is displayed in Table 6.1.

Cotogony	Threshold	Probability of	Event Frequency	Percentile
Calegory	Values	Occurrence	Event Frequency	Trigger
D0	-0.5 to -0.7	30.8%	Once in 3 years	21-30%
D1	-0.8 to -1.2	21.2%	Once in 5 years	11-20%
D2	-1.3 to -1.5	10.7%	Once in 10 years	6-10%
D3	-1.6 to -1.9	5.5%	Once in 20 years	3-5%
D4	-2.0 to - ∞	2.3%	Once in 50 years	0-2%

Table 6.1 SPI probability for drought interpretation

Table 6.2 Original SPI classification used by McKee

SPI Value	Probability of Occurrence	Drought Category
0 to -0.99	50.0%	mild drought
-1.00 to -1.49	15.9%	moderate drought
-1.50 to -1.99	6.7%	severe drought
≤ -2.00	2.3%	extreme drought

It should be noticed that when McKee et al. originally developed the SPI in 1993, they classified the drought intensity differently (Table 6.2.)

To make all the indices comparable, the threshold values for PCI, RAI and SAI are calculated according to the percentile trigger of SPI. That is, the second, fifth, tenth, twentieth, and thirtieth percentiles are used as the threshold values. Tables 6.3 and 6.4 give the threshold values of each of the indices for the Mixed Grassland region and the entire Alberta agricultural region (including Peace Lowland, Boreal Transition, Moist Mixed Grassland, Aspen Parkland and Fescue Grassland/Cypress Hills) for the wheat-growing season (i.e. from April to July.) The PCI here is normalized by $\sqrt{\lambda_1}$.

Catagony	Threshold Values						
Calegory	RAI	PCI	SAI	PCPN (mm)			
D0	-1.2 to -1.7	-0.7 to -0.8	-0.6 to -0.8	179.5 to 164.5			
D1	-1.8 to -2.3	-0.9 to -1.2	-0.9 to -1.0	164.4 to 151.7			
D2	-2.4 to -2.7	-1.3 to -1.4	-1.1 to -1.2	151.6 to 143.5			
D3	-2.8 to -3.3	-1.5 to -1.7	-1.3 to -1.4	143.4 to 129.9			
D4	-3.4 to - ∞	-1.8 to - ∞	-1.5 to - ∞	130.0 to 0.0			

Table 6.3 The threshold values of wheat drought classification for Alberta agricultural region

Table 6.4 The threshold values of wheat drought classification

for the Mixed Grassland region

Category	Threshold Values					
	RAI	PCI	SAI	PCPN (mm)		
D0	-1.3 to -1.7	-0.6 to -0.7	-0.7 to -0.9	138.1 to 124.7		
D1	-1.8 to -2.2	-0.8 to -1.0	-1.0 to -1.2	124.8 to 111.3		
D2	-2.3 to -2.8	-1.1 to -1.3	-1.3 to -1.5	111.2 to 95.5		
D3	-2.9 to -3.4	-1.4 to -1.6	-1.6 to -1.8	95.4 to 80.3		
D4	-3.5 to - ∞	-1.7 to - ∞	-1.9 to - ∞	80.2 to 0.0		

The results show that there are differences of the threshold values for different regions, especially the SAI. This means that there exists regional variance of the indices. The category suitable for one region may not work well for another.

The range of RAI values is

$$(-3\frac{P_m-P}{\overline{m}-\overline{P}},3\frac{P_M-P}{\overline{M}-\overline{P}}),$$

where P_M and P_m are the maximum and minimum precipitations. However, calculating \overline{M} from the ten largest precipitation values and \overline{m} from the ten smallest precipitation values appear arbitrary. Van Rooy (1965) chose 10 because he thought the average of 10 extremes could represent the mean conditions of extremely dry year or extremely wet year. In our study, we consider the event once in 50 years as an exceptional drought (D4) and the event once in 20 years as an extreme drought (D3). We suggest that \overline{M} be the mean of the top 5% precipitation records and \overline{m} be the mean of the bottom 5% precipitation records and \overline{m} , the index will reach -3.0. If 100 years data are used, then \overline{M} is the mean of the largest 5 values. Similarly, \overline{m} is the mean of the smallest 5 values. Tables 6.5 and 6.6 give the comparison of the threshold values for the two different RAI. RAI₁₀ uses the average of the 10 maximum or minimum records as the mean extreme and RAI₅ uses the 5 maximum or minimum records.

Category	Threshold Values			
Category	RAI ₁₀	RAI_5		
D0	-1.2 to -1.7	-1.0 to -1.6		
D1	-1.8 to -2.3	-1.7 to -2.1		
D2	-2.4 to -2.7	-2.2 to -2.4		
D3	-2.8 to -3.3	-2.5 to -2.9		
D4	-3.4 to - ∞	-3.0 to - ∞		

Table 6.5 The threshold values of different RAIs for the Alberta agricultural region

Category	Threshold Values			
category	RAI ₁₀	RAI_5		
D0	-1.3 to -1.7	-1.1 to -1.5		
D1	-1.8 to -2.2	-1.6 to -1.9		
D2	-2.3 to -2.8	-2.0 to -2.5		
D3	-2.9 to -3.4	-2.6 to -3.0		
D4	-3.5 to - ∞	-3.1 to - ∞		

Table 6.6 The threshold values of different RAIs for the Mixed Grassland region

7. Wheat Drought in Canada's Palliser Triangle

The Palliser Triangle is part of the Northern Great Plains and covers southern Alberta, southern Saskatchewan and northern Montana. Because of the barrier provided by the Rockies, the moisture flow from the Pacific Ocean is lifted and cooled, resulting in the dry air mass in southern Alberta (PFRA, 1998).



Fig. 7.1 Historic Prairie wheat drought areas (from PFRA, 1998)

Using a water-budget approach to estimate wheat yield, Williams (PFRA, 1998) mapped the areal extent of 26 annual wheat droughts occurring from 1929 to 1980 in Canada. Six of these drought areas, plus those of 1984 and 1985, are outlined in Fig. 7.1. Figs. 7.2-7.5 demonstrate the comparison of four indices with the precipitation over the Mixed Grassland ecoregion, the southwest corner of Alberta. The April to July precipitation is used to analyze the wheat drought since April-July is the wheat-growing period. The precipitation for the periods of May to August and June to September is also investigated (figures are not included).



Fig. 7.2 April to July precipitation from 1901 to 2000 and corresponding RAI (the solid circles represent the RAI's and the triangles represent the precipitation.)



Fig. 7.3 April to July precipitation from 1901 to 2000 and corresponding SAI(the solid diamonds represent the SAI's and the triangles represent the precipitation.)



Fig. 7.4 April to July precipitation from 1901 to 2000 and corresponding SPI(the solid circles represent the SPI's and the triangles represent the precipitation.)



Fig. 7.5 April to July precipitation from 1901 to 2000 and corresponding PCI. (the solid circles represent the PCI1, the solid diamonds represent the PCI2 and the triangles represent the precipitation.)



Fig. 7.6 Rainfall Decile Index and precipitation (the stars represent the precipitaton.)

The figures show that the indices can capture some of the drought events of 1931, 1936, 1937, 1943, 1961 and 1988. The Rainfall Decile Index (Fig. 7.6) also captured some of the drought events. Those years whose precipitations are below the D2 line are the years with "much below normal" precipitation.

8. Probability Transition of Weekly Precipitation

The drought-wet condition for an EDP is divided into 5 categories according to percentiles of precipitation: extremely dry (0-20 percentile), dry (21-40 percentile), normal (41-60 percentile), wet (61-80 percentile), and extremely wet (81-100 percentile). The probability of the transition from one drought category this week to another in the next week is illustrated as follows (Fig. 8.1).

Fig. 8.2 gives the probability transition for the Mixed Grassland ecoregion. It shows that the chance of transition from extremely dry to extremely wet is the highest in May. The chance of transition from extremely wet to extremely dry is the highest in early February and early October. The chance of transition from normal to extremely dry is the highest in the middle of May. And the chance of transition from normal to dry is the highest in the early January and in the late April. Apparently the transitions from-dry-to-wet and from-wet-to-dry are asymmetric.



Fig. 8.1 The illustration of the probability transition of weekly precipitation.



Fig. 8.2 The probability transition for Mixed Grassland







Fig. 8.3 The 20% to 100% percentiles for five of the ecoregions.

Fig. 8.3 demonstrates the 20, 40, 60, 80, and 100 percentiles for five ecoregions: Aspen Parkland, Mixed Boreal Uplands, Mixed Grassland, Moist Mixed Grassland and Peace lowland/Boreal transition. It shows that in wheat-growing season, the Mixed Grassland has the lowest precipitation and thus is the driest region in Alberta. The data for other five ecoregions have also been computed and are included in the dataset CD.

9. The Gamma Fitting Problem

Gamma fitting is the first step in calculating the SPI index. It is important to know whether or not the gamma distribution can represent the real distribution of precipitation time series. We compared the cumulative probability of the fitted gamma distribution with an empirical cumulative probability for precipitation at different time scales. The empirical cumulative probability used here follows Panofsky and Brier (1958). The precipitation time series is sorted in increasing order, and the empirical cumulative probability (ECP) is defined as

$$ECP = \frac{k}{n+1}$$

where k means the k th observation of the sorted data series and n is the sample size. The monthly, annual and wheat-seasonal (April-July) precipitations for each ecoregion during the period 1901-2000 are used to conduct the gamma fitting. The calculation of the cumulative probability of the fitted gamma distribution is the same as the first two steps of the SPI calculation.

Figs. 9.1-9.3 show the comparison of the empirical cumulative probability and the fitted gamma cumulative probability of precipitation for different time scales over the ecoregion Mixed Grassland. The smooth curve is the cumulative probability of the fitted gamma distribution and the star-marked line is the empirical cumulative probability. The x-axis represents the precipitation value and the y-axis represents the probability value. It can be seen that gamma distribution can reliably represent the precipitation distribution.

However, it is still worthwhile to check whether other distributions, like Pearson III or mixed distribution between lognormal and gamma, can be used to fit the probability distribution function. Guttman (1999) examined the Pearson III distribution and found this distribution sometimes gives a good fit. When studying the ground truth problem for the Tropical Rainfall Measuring Mission of NASA, the mixed distribution was successfully used. Thus, to find out the best fit to a rainfall probability model is certainly worth further investigation and the work is deferred to a later time.



Fig. 9.1 The fitted gamma distribution and the empirical cumulative probability for April-July precipitation.



Fig. 9.2 The fitted gamma distribution and the empirical cumulative probability for annual precipitation.



Fig. 9.3 The fitted gamma distribution and the empirical cumulative probability for April precipitation

10. Conclusions and discussion

Five meteorological drought indices are investigated in this project. The numerical results over the ecoregion Mixed Grassland show that these five indices can capture some of the drought events of Alberta during the 100-year (1901-2000) period. The correlation coefficients show that the drought indices and the precipitation are highly correlated. In the future, further study need to be done on the comparison of the performance of the drought indices. Through our distribution-fitting results for different time scales (monthly to annual), it is found that the gamma distribution can fit the climatological precipitation time series well.

The correlation between each of the four indices and precipitation (Table 10.1) shows that SAI, RAI, SPI and PCI1 are highly correlated with precipitation. Among them, PCI1 has the highest correlation. The correlation of PCI2 (the second principal component) with precipitation is near zero. This result verifies that the first and the second principal components are orthogonal in the EOF computing process.

	SAI	RAI	SPI	PCI1	PCI2	PCPN
SAI	1.0000					
RAI	0.9905	1.0000				
SPI	0.9872	0.9978	1.0000			
PCI1	0.9979	0.9917	0.9877	1.0000		
PCI2	0.0610	0.0062	0.0169	-0.0002	1.0000	
PCPN	0.9987	0.9919	0.9880	0.9998	0.0141	1.0000

Table 10.1 The correlation coefficients between the four indices and the precipitation

The probability transition of weekly precipitation is calculated. It is found that for Mixed Grassland region, the chance of transition from extremely wet to extremely dry is highest in early February and early October; the chance of transition from normal to extremely dry is highest in the middle of May; and the chance of transition from normal to dry is highest in early January and in late April. The robustness of the probability transition needs further investigation.

As mentioned before, snowmelt is an important water supply in Alberta. Snowmelt is related to the temperature, so the temperature should also be considered when analyzing the precipitation deficit. However, determining if the precipitation due to snowmelt is from the current year or from the year before is difficult, so how to relate the temperature to the snowmelt still needed investigation. This statement leads to a question: why meteorological drought indices? Although drought indices in the Palmer class have contained various types of information including precipitation, stream flow, and soil wetness, the complicated parameter estimation in the index computing often make the index insensitive to the drought conditions. On the other hand, the meteorological indices are appropriate in reflecting the balance of the water content of the agricultural soil when the meteorological data are up to certain accuracy. Alberta has a master meteorological dataset that was obtained from optimal interpolation (Shen et al., 2001). The project here would like to take advantage of this dataset. However, the accuracy of the dataset for the mountain regions is still questionable and needs further investigation.

Some future research topics are listed below:

- Optimization method to combine the selected indices: As proposed, the least square approach will be conducted to derive an optimal drought index. The optimal averaging problem will be converted into a problem of computing the covariance matrix of the selected indices. A theoretical breakthrough and significant practical applications are expected.
- 2) Hydrological data: The inclusion of hydrological data in our drought analysis can give us more information about the historical drought events to help with the determination of *I*, the objective drought parameter.
- **3) Further investigation of the drought indices**: The characteristics of the proposed indices will be thoroughly investigated. Their suitability for describing the Alberta drought condition will be studied.
- 4) Area factor in PCI computation: The EOF analyses conducted earlier did not consider the area effect of each polygon. Since the EDP polygons have an irregular shape, the area for each polygon is different. Therefore, the inclusion of area effect is important since the data over a bigger polygon should have a larger weight.
- 5) Spatial pattern of drought event: Analysis of the EOF modes is an effective way to discern the spatial pattern of precipitation, and knowledge of the pattern is useful for drought monitoring and prediction. This pattern also can be incorporated into climate dynamics to explain certain spatial characteristics of drought events.
- 6) Wavelet analysis: Unlike Fourier transformation, which contains only the information about the frequency and intensity of the signal, wavelet transformation can provide local information regarding the time evolution of the signal's spectral characteristics. This information provides a possible way to investigate the frequency, intensity, time position and duration of drought events.
- 7) Related work

a) **Data error**: Although Shen et al. (2001) used cross-validation to evaluate the accuracy of the interpolated data set and showed that the interpolation method can retain not only the climate mean but also the precipitation frequency in a month, data errors still exist because of some bad records and the mismatch of the precipitation frequencies for a given day between the interpolated point and its nearest station. For example, in some cases, the precipitation record of a station for a given day is zero while its nearest station has a large precipitation value. The interpolated value would also be large because the nearest station is used to indicate which day has precipitation.

When the observational stations are sufficiently dense for each day, no such problem would occur unless the station records are wrong or the climate values are highly discontinuous. In our case, the station densities are both spatially and temporally variant because most of the stations have no complete 100-year records. A given point might have a large number of stations around it for one day but a small number another day. Therefore, to evaluate the quality of the interpolated data, station density and the distance to the nearest station should also be taken into account. A proposed index of station density for a grid point j is

$$\rho_j = \left(\sum_{i=1}^{M_j} \frac{1}{d_{ij}}\right)^{-1},$$

where d_{ij} is the distance between *i* th nearest station and the grid point, and M_j is the number of stations used in the interpolation. For the nearest-stationassignment, M_j becomes 1, and ρ_j becomes the distance between the grid point and the nearest station.

b) **Interpolation method improvement**: The interpolation methods we used did not take into account the effect of elevation. Hence, the interpolation accuracy of the mountain area was not considered. A possible way to improve the method is to combine the trivariate thin-plate-spline smoothing method (Hutchinson, 1998) with our hybrid method. The spline interpolation of scattered data involves constructing a thin plate that fits a field with minimum mean square error and satisfies the constraint of continuous curvature. Further experiments need to be done on the topic.

- c) **Prediction of seasonal precipitation:** Shen et al. (2001) developed an optimal ensemble canonical correlation-forecasting model for seasonal precipitation. This model uses a quasi-nonlinear scheme based on the canonical correlation analysis and the empirical orthogonal functions. Experiments showed that it improves prediction skills for seasonal precipitation by a remarkable 10 to 20 percent for all seasons in the United States. This model could be applied to Alberta precipitation forecasting, and the forecasted precipitation could be used as the input data to compute the optimal drought index and hence to predict drought events.
- d) Alberta Agroclimatic Atlas: As an application of the interpolated Alberta climate data set, several data derivatives were calculated to generate the Alberta Agroclimatic Atlas. These include the 30-year normal of annual total precipitation, the 30-year normal of May 1 to August 31 total precipitation, the frost-free period, and the length of the growing season. The results can provide an intuitive impression of Alberta's climatic characteristics and will help farmers and government officials to understand drought events.

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